Sum, Difference and Shaped Beam Pattern Synthesis by Non-Uniform Spacing and Phase Control

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***Abstract*—A design procedure for the synthesis of non-uniformly spaced linear arrays is presented, which uses the Poisson sum expansion of the array factor introduced in the literature. By considering the nonzero phase term in the existent formula and using the appropriate line source pattern synthesis methods, a general design procedure is obtained to synthesize any type of pattern, such as sum, difference and shaped beams. This approach converts the nonlinear complex problem of pattern synthesis for non-uniformly spaced linear arrays to a simple problem, which makes it fast and easy to implement. Moreover, an extra optimization process is added to the synthesis procedure to improve final pattern and provide control on computed parameters.**

***Index Terms*—Linear arrays, non-uniformly spaced arrays.**

I. Introduction

APPLICATION of non-uniformly spaced linear arrays instead of uniformly spaced linear arrays has been an attractive topic in antenna area for some years. Simplicity of the feeding network for non-uniformly spaced linear arrays is the most important feature of this type of array. Unz [1] in 1960 analyzed and developed a matrix formulation for non-uniformly spaced linear arrays. Harrington [2] developed an iterative method to reduce the sidelobe level of uniformly excited N-element linear array to about 2/N times the field intensity of the mainlobe by employing unequal spacings. Since the element positions occur in the argument of exponential function, the desired pattern synthesis by proper calculation of element positions is a nonlinear and complex problem. Regarding this fact, most of the previous works have attacked the problem by utilizing numerical and computer-based iterative [3] or stochastic optimization methods [4]–[6]. The development of non-uniform fast Fourier transform (NFFT) and its applications for sidelobe level reduction are among the current interesting research activities [7]. In most of these works, the main goal is only the sidelobe level reduction and they are not able to synthesize patterns with arbitrary shape. Although, there is a lack of analytical formulas for this type of array, yet in recent years scarcely any work is reported in this area [8], [9]. But the first well-structured mathematical formulation was introduced by Ishimaru [10], who used the Poisson sum expansion of the array factor and some simplifying assumptions to derive a relationship between the pattern and element spacings. He used the derived relation to reduce the sidelobe level.

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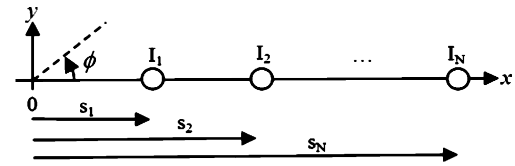

Fig. 1. Linear array with arbitrarily spaced elements.

This paper will consider a nonzero phase term in the Ishimaru's formula to make it capable of synthesizing any type of pattern, such as sum, difference and shaped beams by non-uniform spacings and/or element phase control. The phase term allows the synthesis of asymmetrical sum pattern (i.e., sum pattern with sidelobes asymmetrically placed around the mainlobe) by non-uniform spacings and phase control which have not been addressed up to now. The proposed method utilizes the derived formulas to convert the nonlinear complex problem to a relatively simpler one (namely the line source pattern synthesis), and solve it by introducing an appropriate line source pattern synthesis method. This method is fast and easy to implement, despite the fact that the synthesis of patterns of any shape by non-uniform spacings and also element phase control (if necessary) is a complex problem and has not been considered in the literature. Furthermore, an additional genetic algorithm (GA)-based optimization method is introduced to improve the synthesized pattern and provides a control on the calculated phase and/or spacing. This extra GA-based optimization process is necessary for the shaped beam pattern synthesis.

II. Theoretical Background

A. Poisson Sum Expansion of Array Factor

Assume a linear array of antennas composed of arbitrarily spaced N elements on the x-axis as shown in Fig. 1. Its array factor can be defined as

$$E(\phi)=\sum_{n=1}^{N} I_n e^{jks_n\cos\phi} \tag{1}$$

where I_n is the n'th element current excitation, k is the wave number, s_n is the position of n'th element relative to the origin, ϕ is the angle relative to x-axis, and E is the array factor.

Here, the Ishimaru's results from [10] are briefly summarized, and then a general case will be considered.

Ishimaru shows that by good approximation for not-very-long arrays, the array factor using the Poisson's sum formula can be written as [10]

$$E(u) \approx E_0(u) \tag{2}$$

$$E_0(u) = \left(\int_{-1}^{1} (dy/dx) e^{j\pi ux} \right) \Big/ 2 \tag{3}$$

where $x = x(y)$ is the normalized source position function $(-1 < x < 1)$ and $y = y(x)$ is normalized source number function $(-1 < y < 1)$. Also, u is defined as

$$u = 2a \cos\phi/\lambda \tag{4}$$

where a is $(s_N - s_0)/2$. The actual position of nth element is

$$s_n = a x(y_n) \tag{5}$$

where the value of y_n is provided in (20) and (21) of [10].

By defining $f(x)$ as

$$f(x) = dy/dx \tag{6}$$

equation (3) can be written as [10]

$$E_0(u) = (1/2). \int_{-1}^{1} f(x) e^{j\pi ux} dx. \tag{7}$$

Assuming a line source with amplitude of excitation equal to $f(x)$, then (7) provides its pattern. In the derivation of (7), it is assumed that the excitation of elements is equal to unity with zero phase. Here, we use a more general case by assuming that the element excitations have unit magnitude (to avoid feeding complexity) but nonzero phase as $1\angle\alpha_n$ (to have more control on pattern). By defining $\beta(x)$ as a continuous version of α_n, (7) is modified to

$$E_0(u) = (1/2). \int_{-1}^{1} f(x) e^{-j\beta(x)} e^{j\pi ux} dx. \tag{8}$$

Consequently, the nonlinear problem (pattern synthesis by non-uniform spacing and phase control) has been converted to finding $f(x)$ and $\beta(x)$ which are similar to the current distribution for line sources. Although (8) is reported in [10], but later, the phase term is assumed to have no variation in order to use available line source pattern synthesis methods. Here, a nonzero phase term will be used which leads to a new position/phase synthesis process instead of only position synthesis. To clarify the role of phase term, it should be noted that in the line source case, based on the Fourier analysis, the excitation current is always real and symmetrical around the center for symmetrical sum pattern [11]. But, for the asymmetrical sum pattern only the amplitude of current distribution is symmetrical around the center and the phase distribution is asymmetrical and non-zero [11]. Regarding this fact and referring to (3), a complex value will be obtained for positions of elements to synthesize asymmetrical sum patterns which is unacceptable. But, using the exponential term $\beta(x)$ in (8), one can generate real and symmetrical values for $f(x)$, even for asymmetrical patterns. This point of view has not been addressed and no attempt has been made to synthesize asymmetrical sum patterns with arbitrarily specified sidelobe levels by non-uniform spacings and phase control.

B. Desired Pattern Synthesis With Pattern's Null Manipulation

In order to synthesize the desired pattern, $f(x)$ and $\beta(x)$ should be evaluated using line source pattern synthesis methods. Elliott's method [11], as a well-known method, uses a null perturbation process to obtain some control on individual sidelobes, but his method has no control on the filled nulls. Here, Elliot's basic ideas for the manipulation of nulls is applied, but by using GA. Also, the complex valued nulls are used instead of real valued nulls for the synthesis of shaped beam patterns. This method has control on both the deep and filled nulls as well as sidelobe levels and filled null areas (shaped beam areas).

In the following, for each pattern type, appropriate pattern null manipulation method is introduced.

1) Sum Pattern: For the sum patterns a generic pattern is defined as follows [11]:

$$G_{\text{sum}}(u) = \sin\pi u/\pi u. \tag{9}$$

To have full control on sidelobes, one should manipulate nulls of this generic pattern as follows [11]:

$$E(u) = \frac{\sin\pi u}{\pi u} \frac{\prod_{n=-(\bar{n}_L-1), n\neq 0}^{(\bar{n}_R-1)} (1 - u/u_n)}{\prod_{n=-(\bar{n}_L-1), n\neq 0}^{(\bar{n}_R-1)} (1 - u/u_n)} \tag{10}$$

where R and L show the right and left hand sides of the mainlobe for u as variable, respectively [11]. The right side includes $0 \leq \phi < 90^\circ$ and the left side includes $90^\circ < \phi \leq 180^\circ$. Also, $\bar{n}_R - 1$ is the number of displaced nulls in the right hand side and $\bar{n}_L - 1$ is the number of displaced nulls in the left hand side. If nulls (u_n) can be calculated, the Fourier expansion method as applied in [11] can be used to write $f(x)$ and $\beta(x)$ as

$$f(x) e^{-j\beta(x)} = \sum_{m=-\infty}^{\infty} B_p e^{-jm\pi x} \tag{11}$$

and then substituting (11) in (8) gives

$$\begin{aligned} E_0(u) &= (1/2). \int_{-1}^{1} \sum_{m=-\infty}^{\infty} B_m e^{-jm\pi x} e^{j\pi ux} dx \\ &= (1/2). \sum_{m=-\infty}^{\infty} B_m \int_{-1}^{1} e^{-jm\pi x} e^{j\pi ux} dx \end{aligned} \tag{12}$$

where, the integral in (12) for $u \neq m$ is zero and for $u = m$ reduces to

$$E_0(m) = B_m. \tag{13}$$

Since $E_0(p)$ is zero for $m \geq \bar{n}_R$ or $m \leq -\bar{n}_L$, then applying (13) to (11) leads to a truncated Fourier series as

$$f(x)e^{-j\beta(x)} = \sum_{m=-(\bar{n}_L-1)}^{(\bar{n}_R-1)} E(m)e^{-jm\pi x} \tag{14}$$

$f(x)$ and $\beta(x)$ are amplitude and phase of the right side summation, respectively.

Now, we need to calculate u_n. First u_n is defined as

$$u_n = \overset{0}{u}_n + \delta u_n \tag{15}$$

where u_n is the n'th null of generic pattern, which is used as its initial value, and δu_n is the displacement of n'th null to be computed here by GA. More details about the cost function definition and the parameters of applied GA are provided in Appendix.

2) Difference Pattern: For the difference pattern, a generic pattern is defined as follows [11]:

$$G_{\text{Diff}} = \frac{\pi u \cos(\pi u)}{(u-1/2)(u+1/2)}. \tag{16}$$

Again for full control, the nulls may be displaced by the following relation [11]:

$$E(u) = \pi u \cos(\pi u) \frac{\prod_{n=-(\bar{n}_L-1), n\neq 0}^{(\bar{n}_R-1)} (1-u/u_n)}{\prod_{n=-(\bar{n}_L)}^{(\bar{n}_R-1)} (1-u/(n+1/2))} \tag{17}$$

where u_n may be defined as (15) and GA is used for their calculation. After the appropriate null calculations (see Appendix), $f(x)$ and $\beta(x)$ can be derived as

$$f(x)e^{-j\beta x} = \sum_{m=-(\bar{n}_L-1)}^{(\bar{n}_R-1)} D(m+1/2)e^{-j(m+1/2)\pi x} \tag{18}$$

where $D(m+1/2)$ is defined as [12]

$$D(m+1/2) = (-1)^m (m+1/2)\pi. \quad 0 \leq m \leq \bar{n}_R - 1$$
$$\cdot \frac{\prod_{n=-(\bar{n}_L-1)}^{(\bar{n}_R-1)} (u_n - (m+1/2))}{\left[\prod_{n=0,n\neq m}^{(\bar{n}_R-1)} \left(1-\frac{m+1/2}{n+1/2}\right)\right]\left[\prod_{n=0}^{(\bar{n}_L-1)} \left(1+\frac{m+1/2}{n+1/2}\right)\right]} \tag{19}$$

$$D(m+1/2) = (-1)^m (m+1/2)\pi. \quad -\bar{n}_L \leq m \leq -1$$
$$\cdot \frac{\prod_{n=-(\bar{n}_L-1)}^{(\bar{n}_R-1)} (u_n - (m+1/2))}{\left[\prod_{n=0,n\neq m}^{(\bar{n}_R-1)} \left(1-\frac{m+1/2}{n+1/2}\right)\right]\left[\prod_{n=0}^{(\bar{n}_L-1)} \left(1+\frac{m+1/2}{n+1/2}\right)\right]}. \tag{20}$$

3) Shaped Beam Pattern: To generate this type of pattern, the generic pattern introduced for the sum pattern may be used, but here complex nulls ($u_n + jp_n$'s) should be introduced and manipulated as [13]

$$E(u) = \frac{\sin \pi u}{\pi u} \frac{1}{\prod_{n=1}^{\bar{n}+\text{MF}-1} (1-u^2/n^2)} \left[\prod_{n=1}^{\text{MF}} (1-u^2/(u_n+jp_n))\right]$$
$$\cdot \left[\prod_{n=1}^{MF} (1-u^2/(u_n-jp_n))\right] \prod_{n=MF+1}^{\bar{n}-1} (1-u^2/n^2) \tag{21}$$

where $\bar{n}-1$ shows the number of displaced deep nulls and MF shows the number of filled nulls. For this case, $f(x)$ and $\beta(x)$ are derived as [13]

$$f(x)e^{-j\beta x} = E(0) + 2\sum_{m=1}^{\bar{n}+\text{MF}-1} E(m) \cos m\pi x. \tag{22}$$

C. Calculation of Position and Phase for Each Element

Applying the computed $f(x)$ from (22) to (6) and solving the equation, we have

$$y(x) = \frac{\int_0^x f(x)dx}{\int_0^1 f(x)dx}. \tag{23}$$

The integrations may be done by the trapezoidal rule. For the symmetrical pattern, some analytical relations may be obtained. Then, the function $x(y)$ may be derived by analytical inversing techniques or geometrical method. Finally, the appropriate positions of elements x_n will be calculated from $x_n = x(y_n)$, using (20) or (21) of [10]. Having the positions then $\beta(x)$ can be sampled to yield α_n.

III. The Application of Proposed Synthesis Method

Here, with some examples the capability of the proposed method to synthesize different types of patterns is demonstrated.

A. Sum Pattern Synthesis

Here, an asymmetrical sum pattern with the mainlobe at $\phi = 90°$ (i.e., broadside pattern) is synthesized. Three innermost sidelobes (i.e., the first three sidelobes which are in the vicinity of mainlobe) for $0 \leq \phi < 90°$ are set to be lower than -31 dB and the other sidelobes in this region are set to be under -20 dB. For the second half of the pattern, $90° < \phi \leq 180°$, a shape similar to the pattern of uniformly excited array is specified. The number of elements is $N = 27$. As the first step, a line source pattern should be synthesized. The GA (with the cost function explained in Appendix) manipulated the line source pattern's nulls and found their appropriate positions to give the desired pattern. Then, $f(x)$ and $\beta(x)$ are calculated by (14). By applying $f(x)$ to (23) and performing the integration, $y(x)$ is calculated. After the computation of $y(x)$, the values of x_n are calculated and then by sampling $\beta(x)$ at x_n, the element phases α_n are derived. The achieved pattern for the array degrades from the

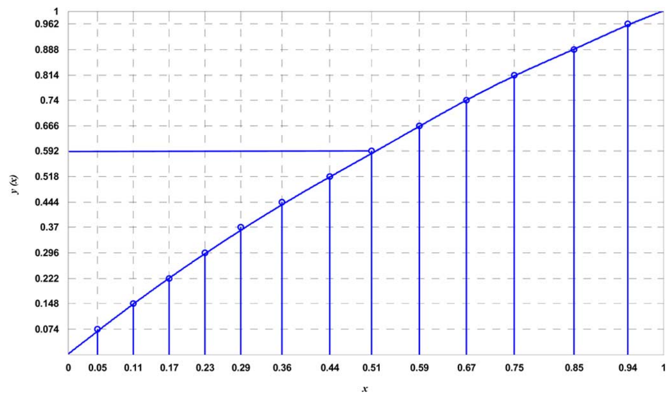

Fig. 2. The function $y(x)$ relating the position of elements with their number is shown for synthesized pattern in Fig. 3.

TABLE I
SPACING AND PHASE OF LEFT SIDE ELEMENTS OF DESIGNED ARRAYS

Sum pattern			*Difference pattern*			*Shaped beam*		
n	s_n/λ	α_n*(deg)*	*n*	s_n/λ	α_n*(deg)*	*n*	s_n/λ	α_n*(deg)*
1	-3.83	-37.75	1	-5.73	0	1	-3.61	0
2	-3.44	-26.26	2	-5.15	0	2	-2.44	180
3	-3.06	8.34	3	-4.73	0	3	-1.87	180
4	-2.71	15.28	4	-4.41	0	4	-1.22	0
5	-2.40	10.52	5	-4.13	0	5	-0.76	0
6	-2.10	7.00	6	-3.88	0	6	-0.66	0
7	-1.79	4.41	7	-3.65	0	7	-0.53	0
8	-1.49	2.31	8	-3.43	0	8	-0.40	0
9	-1.21	1.02	9	-3.22	0	9	-0.24	0
10	-0.94	0.34	10	-3.02	0	10	-0.17	0
11	-0.69	0.55	11	-2.82	0	11	-0.06	0
12	-0.45	1.11	12	-2.62	0			
13	-0.22	0.98	13	-2.43	0			
14	0	0.00	14	-2.23	0			
			15	-2.02	0			
			16	-1.81	0			
			17	-1.57	0			
			18	-1.31	0			
			19	-1.01	0			
			20	-0.57	0			

line source pattern. The same event happens when one tries to uniformly sample the continuous line source distribution to feed a discrete array, when it varies a lot with position [11]. For uniformly spaced arrays, the uniform sampling theory determines the required minimum number of elements and maximum element spacing. But for non-uniformly spaced linear arrays, the situation is complex and the well-known uniform sampling theory will not work. The definition of average element spacing in [6], as $s_{\text{avg}} = (\max(s_n) - \min(s_n))/(N-1)$, enables us to apply partly uniform sampling criteria to non-uniformly spaced linear arrays. One can treat the average element spacing like that in uniformly spaced linear arrays and set a constraint on it to satisfy the uniform sampling conditions. But this is not a general solution and one can start the pattern synthesis with the proposed method and then based on the results, take an additional modification procedure. This additional modification may be either an easy manipulation (e.g., by trial and error) to change the initial line source pattern to achieve the final desired pattern or an optimization procedure that directly varies the element spacings to find their optimum values. This additional modification is comparable to the iterative procedure reported in [11] to improve the uniformly spaced array pattern using sampled line source data. For the current example, just simple manipulation of the sidelobe levels of initial line source pattern could help. Three trial and errors showed that in the interval $90° < \phi \leq 180°$, the line source pattern should be set about 5 dB below the desired value (e.g., about -20 dB), in order to have the desired pattern for non-uniformly spaced array. Using this line source, $y(x)$ is calculated and drawn in Fig. 2 (due to the symmetry only the right side is depicted). The element spacings and their phases are listed in Table I (on the left section). In this paper, always the first element on the left side is numbered one, and the last one on the right side is N. The pattern of the synthesized array is shown in Fig. 3. Also, for comparison, the used line source pattern (after modification) is shown in Fig. 3. The CPU time on a 2 GHz Celeron PC was about 50 seconds (about 90% of this time was taken up by the GA run).

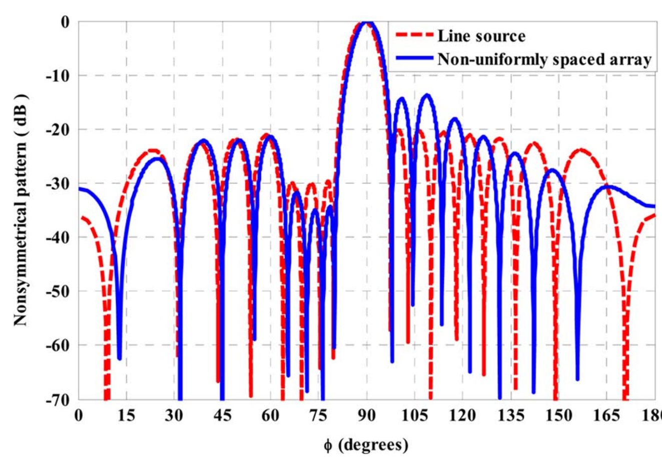

Fig. 3. The synthesized asymmetrical pattern by non-uniform spacing and phasing in comparison with initially used line source pattern.

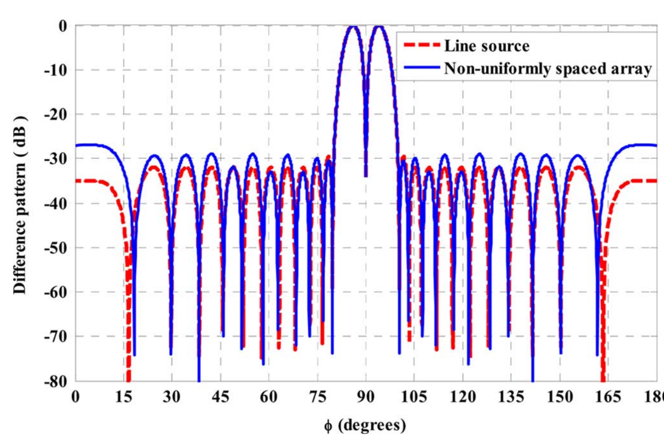

Fig. 4. The synthesized difference pattern by non-uniform spacing and phasing in comparison with initially used line source pattern.

B. Difference Pattern Synthesis

Here, for a 40-element array, a pattern with sidelobe level of -27 dB is synthesized. Applying GA to (17) and following the same procedure as for the sum pattern synthesis, the position and phase of each element are calculated and listed in Table I (middle section). Phase of right side elements are 180°. As compared in Fig. 4, the pattern of line source and non-uniformly

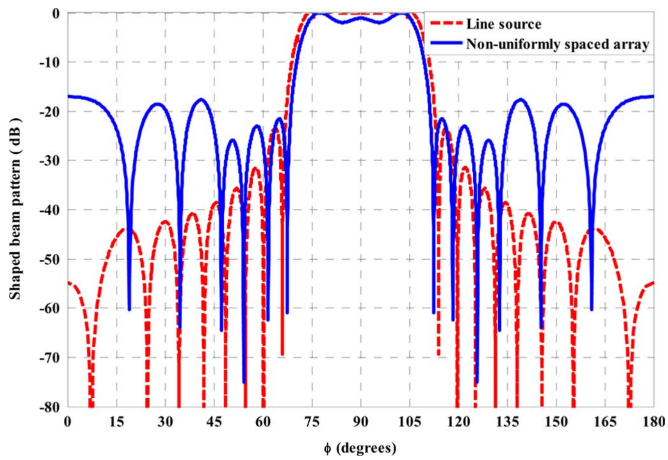

Fig. 5. The synthesized flat top pattern by uniform excitation and non-uniform spacing in comparison with initially used line source pattern.

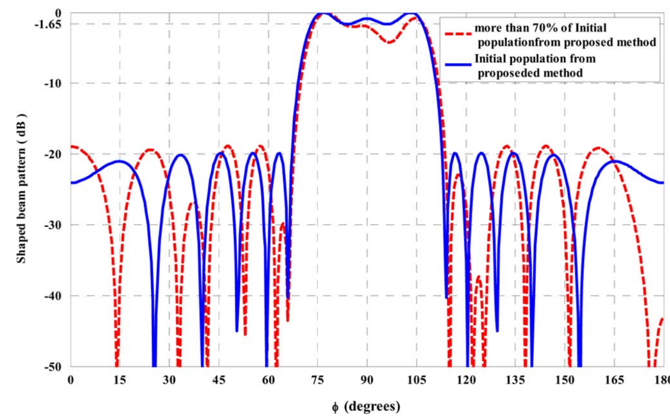

Fig. 6. The modified synthesized pattern in Fig. 5 by extra optimization in comparison with the synthesized pattern from beginning using GA with phase data from proposed method but 30% of value of positions are changed.

spaced array are close enough. The CPU time for its design on the same PC is less than 30 seconds, and again 90% of this time is consumed by GA.

C. Shaped Beam Pattern Synthesis

Although, there are some works for the synthesis of shaped beam patterns such as $\csc^2$ and flat top patterns by calculating the phases and keeping the amplitudes constant or almost constant [14], [15], the non-uniform spacing may be seen as another candidate to synthesize these patterns. Here, by an example, we show that the proposed synthesis procedure can be used for this purpose. In this example, it is desired to have a flat top pattern with ripples of 2 dB centered on -1 dB and sidelobe level of -20 dB by an $\mathrm{N} = 22$ element array. Applying GA to (21) and using the described procedure, the optimum position of each element is calculated in less than 45 seconds. The synthesized pattern is drawn in Fig. 5. Due to the complexity of this type of pattern, the synthesized pattern degrades from the initially used line source. To overcome this problem, an extra optimization stage is added.

In this procedure, the derived element positions ($\overset{0}{s}_n$) are a little bit disturbed by defining, $s_n \ \overset{0}{s}_n + \delta s_n$ and using GA to find the appropriate δs_n to provide the desired pattern. Since, we are close to the optimum positions in the initial approach and so a limited variation is needed, then the GA algorithm converges very fast. The cost function for the current example is as

$$C_{\mathrm{whole}} = C_{\mathrm{sll}}(\delta s_n) + C_{\mathrm{sb}}(\delta s_n) \tag{24}$$

where $C_{\mathrm{sll}}(\delta s_n)$ is the cost function for sidelobe level and is defined on the sidelobe region (SR) as

$$C_{\mathrm{sll}} = (\delta u_n) = |\max(E(\phi))|_{\phi \in \mathrm{SR}} - \mathrm{SLL}_d|^2 \tag{25}$$

also, $C_{\mathrm{sb}}(\delta s_n)$ is defined on the shaped beam region (SB) as,

$$C_{\mathrm{sb}} = C_{\mathrm{sb},1} + C_{\mathrm{sb},2} \tag{26}$$

where $\mathrm{C}_{\mathrm{sb},1}$ for $|\min(E(\phi))|_{\phi \in \mathrm{SB}} - \mathrm{DL}| > \mathrm{RP}$ is

$$C_{\mathrm{sb},1} = |\min(E(\phi))|_{\phi \in \mathrm{SB}} - \mathrm{DL}|^2 \tag{27}$$

and it is zero elsewhere. Also, for $|\max(E(\phi))|_{\phi \in \mathrm{SB}} - \mathrm{DL}| > \mathrm{RP}$

$$C_{\mathrm{sb},2} = |\max(E(\phi))|_{\phi \in \mathrm{SB}} - \mathrm{DL}|^2 \tag{28}$$

and it is zero elsewhere. Moreover, DL is the desired pattern level in SB region (here, $\mathrm{DL} = -1$ dB and $\mathrm{RP} = 2$ dB). Also, for transition from SB to SR a monotonic pattern condition is imposed. Adding this new step to the design procedure, the calculated element positions are listed in the right section of Table I and the pattern is drawn in Fig. 6. The sidelobe level is -20 dB and the ripple around -1 dB is less than 1 dB which complies with the design goals. The CPU time for this new step is about 1 minute for convergence to the desired pattern. This extra optimization can apply the constraint on spacings and phases of elements and so provides more control on the design values. It is interesting to mention that the extra optimization procedure was fed by the new different initial values (different initial population). First, the phase was kept the same as before but 30% of element positions were changed. The optimizer (GA) could converge to an acceptable pattern in 30 minutes (for the initial population size of 400 and number of generations of 40). This pattern is shown in Fig. 6.

Later, GA was fed with randomly valued initial phases and positions for all elements. For this case, after 10 hours with trying many different cost functions, GA did not converge to any acceptable answer. This shows that using GA to design the array from beginning does not work for complex patterns and also shows the important role of the proposed first procedure for approximate position/phase control.

IV. Conclusion

In this paper, the Ishimaru's formulas are used with nonzero value for element's excitation phase in order to synthesize any type of pattern, such as sum, difference and shaped beams. The array pattern synthesis is performed by non-uniform spacing and if necessary by phase control. Furthermore, an extra optimization procedure was applied on calculated values to modify them in order to improve the final pattern. This procedure provides more control on the spacings and phases by possibly applying constraints on them. The proposed method is very fast and easy to implement.

APPENDIX
COST FUNCTION DEFINITION AND FEATURES OF USED GA

To synthesize a desired line source pattern, appropriate positions for nulls should be determined. To perform this, new positions of nulls are assumed to be related to initial values (generic pattern nulls) as (15) and with good accuracy one can assume sidelobes of array pattern to occur between two neighboring nulls as

$$u_m^p = |(u_m + u_{m+1})/2$$
$$m = -N_n, -(N_n - 1), \ldots, N_n - 2, N_n - 1, m \neq 0 \quad \text{(A1)}$$

where $N_n = [2a/\lambda]$. For the sum pattern case, at $u_0^p = 0$ the pattern has a peak but for the difference pattern a null is located there. Therefore, the sidelobe level can be calculated as

$$\mathrm{SLL}_m = (E(u_m^p)). \quad \text{(A2)}$$

Now, an appropriate cost function can be defined based on the design specifications. In general, the problem is to synthesize a pattern with its sidelobes individually adjusted to a desired value $(\mathrm{SLL}_{m,d})$. Then, a cost function can be as

$$C_{\mathrm{sll}}(\delta u_n) = \sum_{m=1}^{N_{\mathrm{sl}}} |\mathrm{SLL}_m - \mathrm{SLL}_{m,d}|^2 \quad \text{(A3)}$$

where N_{sl} is the number of sidelobes. For the shaped beam pattern case, appropriate complex nulls should be calculated to achieve the desired pattern. In order to do this, nulls are manipulated as

$$u_n + jv_n = \overset{0}{u}_n + \delta u_n + j(\overset{0}{v}_n + \delta v_n). \quad \text{(A4)}$$

As it is explained for (21), MF filled nulls $(\delta v_n \neq 0)$ and $\bar{n} - 1$ deep nulls $(\delta v_n = 0)$ should be calculated to minimize the following cost function:

$$C_{\mathrm{whole}} = C_{\mathrm{sll}}(\delta u_n) + C_{\mathrm{sb}}(\delta u_n, \delta v_n) \quad \text{(A5)}$$

where $C_{\mathrm{sll}}(\delta u_n)$ is the cost function for sidelobe level and is defined in the sidelobe region (including deep nulls) and is the same as (A3). Also, $C_{\mathrm{sb}}(\delta u_n, \delta v_n)$ is defined in the shaped beam region (including filled nulls) as,

$$C_{\mathrm{sb}}(\delta u_n, \delta v_n) = \sum_{n=1}^{\mathrm{MF}} C_{\mathrm{RP},n} \quad \text{(A6)}$$
$$C_{\mathrm{RP},n} = \| E(u_n + jv_n) - E(u_n^p)| - \mathrm{RP}_d|^2 \quad \text{(A7)}$$

where RP_d is the desired ripple factor and

$$u_n^p = (u_n + u_{n+1})/2. \quad \text{(A8)}$$

GA as a robust optimizer was selected to minimize the cost functions defined in (A3) and (A5). We set the crossover function as scattered, and mutation as adaptive. Number of population was 65 and number of generations was 20. Also, since the inverse function calculation is very fast, the CPU run time for GA is approximately equal to the time reported earlier for each example.

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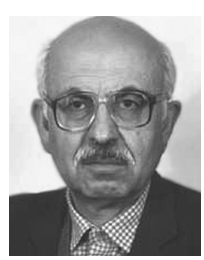

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